INTRODUCTION TO COMPUTER MUSIC
PHYSICAL MODELS

Roger B. Dannenberg

Professor of Computer Science, Art, and Music

Introduction

- Many kinds of synthesis:
  - Mathematical functions (FM, Additive)
  - Sampling
  - Source/Filter models
- None model complexities of physical systems
- When aspects of physical systems defy analysis, we can resort to simulation
- Even simulation is selective, incomplete
- Key is to model the interesting aspects while keeping the simulation computation tractable

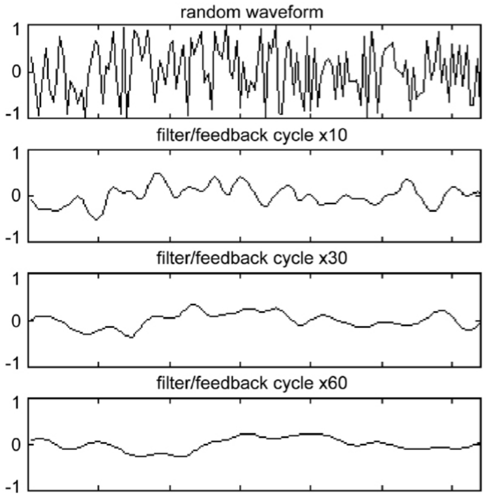
Mass-Spring Model of a String

- Expensive to compute
  - But computers are fast
  - Discrete time simulation is mostly multiplies and adds
- Number of modes (partials) corresponds to number of masses.
- Can add stiffness and other interesting properties

A Variation – Karplus-Strong Plucked String Algorithm

- Fill table with noise or initial conditions
- Perform table-lookup oscillator on noise
- Phase-increment = 1
- Average adjacent samples as they are read
  - Averaging adjacent samples is a low-pass filter
  - Averaging causes global exponential decay
- Very efficient simulation of string behavior

Karplus-Strong (2)



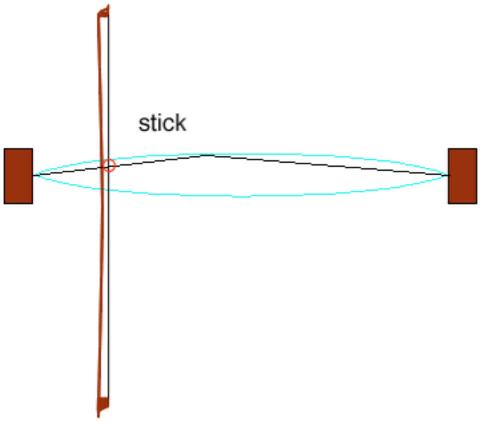
http://music.columbia.edu/cmc/musicandcomputers/chapter4/04_09.php

Improving Karplus-Strong

- Problem: integer table lengths
- Solution: all-pass filter with fractional delay
- Problem: changing string length
- Solution: interpolate all-pass filter
- Problem: controlling decay, loss
- Solution: use different filter (than averaging)

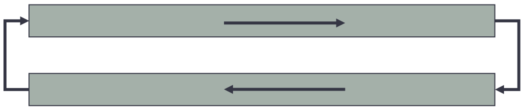
Mechanical Oscillator

- http://www.phys.unsw.edu.au/jw/Bows.html



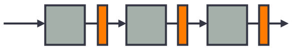
Waveguide Model

- Introduced by Julius Smith
- Wave propagation modeled by delay
- Left-going and right-going waves are separate
- Physical variable (amplitude or flow) is *sum* of corresponding values in two delay lines



"Lumped" Filters

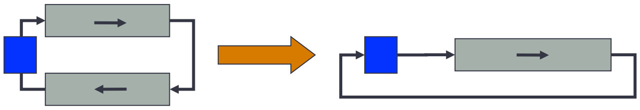
- Real systems (transmission lines, strings, air columns) exhibit continuous, distributed losses
- Length (therefore period) can be frequency-dependent
- Can model losses within waveguide:



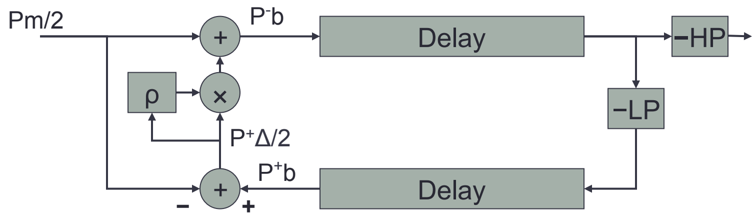
- Or, "lump" losses at the end for efficiency:

McIntyre, Woodhouse (1979), + Schumacher (1983)

- Physicists trying to understand the nature of oscillation in acoustical instruments
- Model:
  - Delay-line loop of one period
  - Low-pass filter modeling losses over one loop
  - Non-linear element to generate oscillation



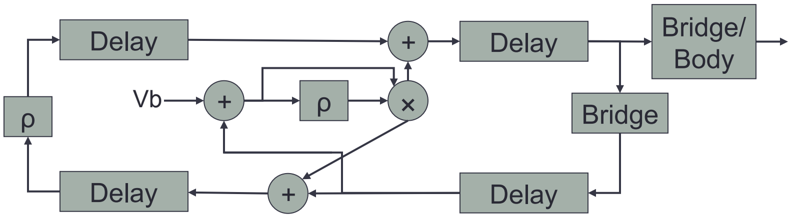
Smith: Efficient Reed-Bore and Bow-String Mechanisms (ICMC 86)



Mouth Reed Bore Bell

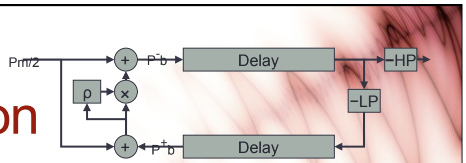
Pm/2 = mouth pressure, ρ(P⁺Δ/2) = reflection coefficient
(lookup table)

Bowed String Model



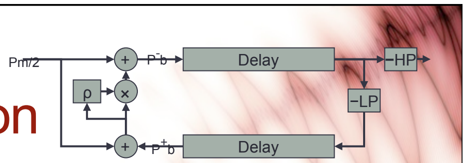
Nut String Bow String Bridge Body

Here, delays contain velocity rather than pressure



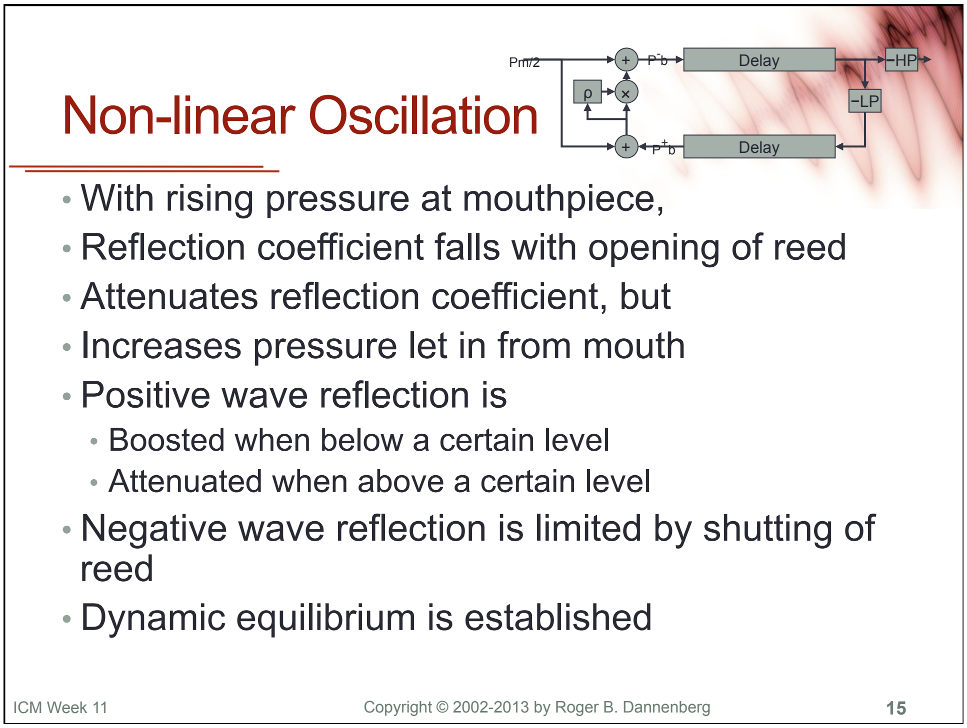
Non-linear Oscillation

- Apply pressure – biases reed to “negative resistance”
- High pressure front to bell, reflects as negated front
- Negated front returns and reflects again (no sign inversion because mouthpiece is approximately closed, not open)
- Negative pressure zone is left behind
- Reflection from open end again brings return-to-zero wave traveling back to mouthpiece
- Positive traveling wave reaches mouthpiece and starts second period of oscillation



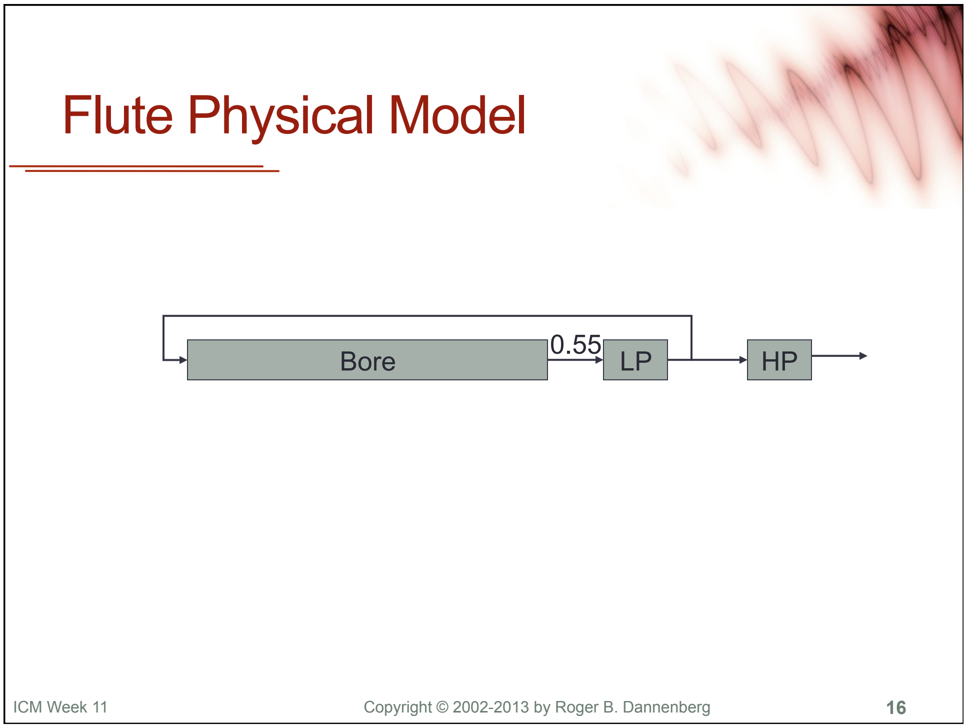
Non-linear Oscillation

- There are losses, so we need to feed energy in
- When pressure drop reflects from mouthpiece, mouthpiece switches from high to low pressure
- Reed changes from open to closed
- Closing increases reflection coefficient and amplifies reflection (with maximum gain of 1)
- Also shuts off pressure coming from mouth – potential gain is greater than 1.



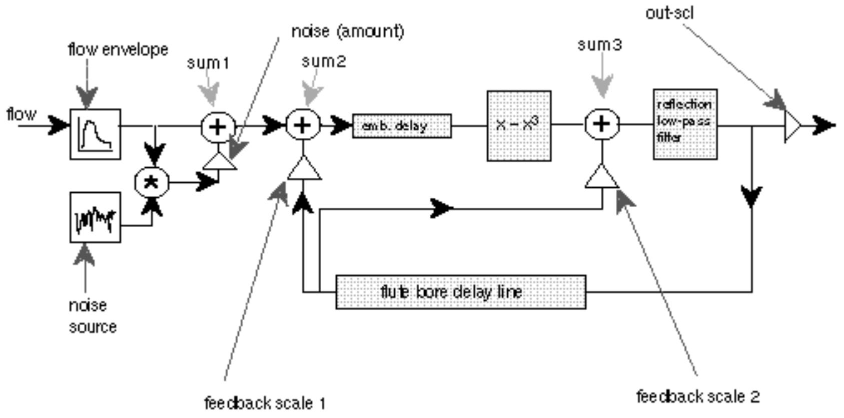
Non-linear Oscillation

- With rising pressure at mouthpiece,
- Reflection coefficient falls with opening of reed
- Attenuates reflection coefficient, but
- Increases pressure let in from mouth
- Positive wave reflection is
  - Boosted when below a certain level
  - Attenuated when above a certain level
- Negative wave reflection is limited by shutting of reed
- Dynamic equilibrium is established



Flute Physical Model

Detailed Diagram



Physical Models in Nyquist

```
(pluck pitch [dur] [final-amp])
```

Variations on STK clarinet model:

```
(clarinet step breath-env)
(clarinet-freq step breath-env freq-env)
(clarinet-all step breath-env freq-env vibrato-freq
  vibrato-gain reed-stiffness noise)
```

Variations on STK saxophony model:

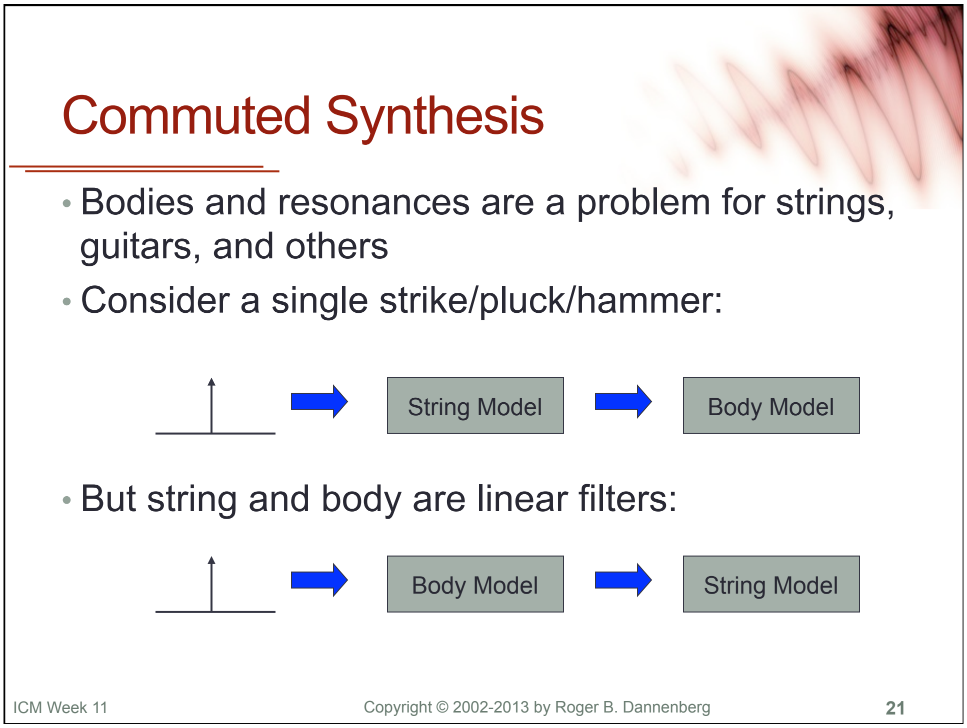
```
(sax step breath-env)
(sax-freq step breath-env freq-env)
(sax-all step breath-env freq-env vibrato-freq
  vibrato-gain reed-stiffness noise blow-pos reed-
  table-offset)
```

More Physical Models in Nyquist

- See manual for more.

MORE PHYSICAL MODELS

Commuted Synthesis
Electric Guitar Model
Analysis
2D Waveguide Mesh

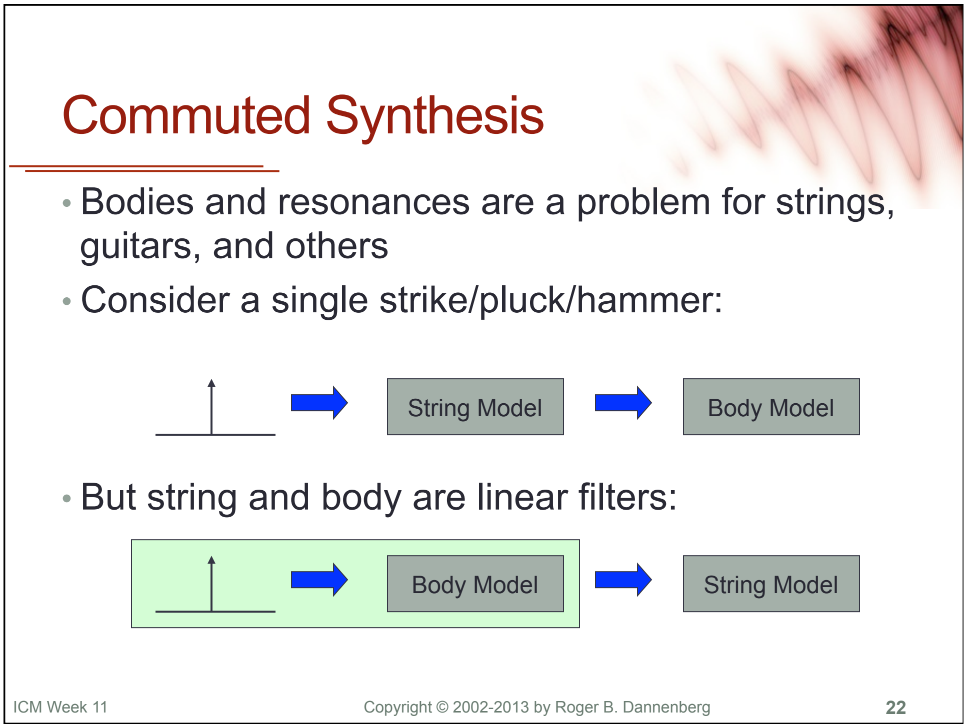


Commuted Synthesis

- Bodies and resonances are a problem for strings, guitars, and others
- Consider a single strike/pluck/hammer:

- But string and body are linear filters:

ICM Week 11



Commuted Synthesis

- Bodies and resonances are a problem for strings, guitars, and others
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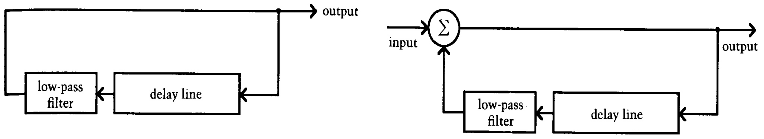
ICM Week 11

Commuted Synthesis

- So, drive the string with impulse response of body
- When bow slips on string, it generates a sort of impulse
- At every bow slip, insert body impulse response into string model
- Good model for piano synthesis, where
  - driving force is simple (hammer hitting string)
  - body is complex (sound board)

Electric Guitar (Charles R. Sullivan)

- Extending Karplus Strong…



- Low-pass filter
  - Determines decay rate
  - Would like to control it at different frequencies
  - FIR filter: $y_n = a_0x_n + a_1x_{n-1} + a_2x_{n-2}$
  - Problem: potentially has gain >= 1 at zero Hz (DC)

Loop Filter Design

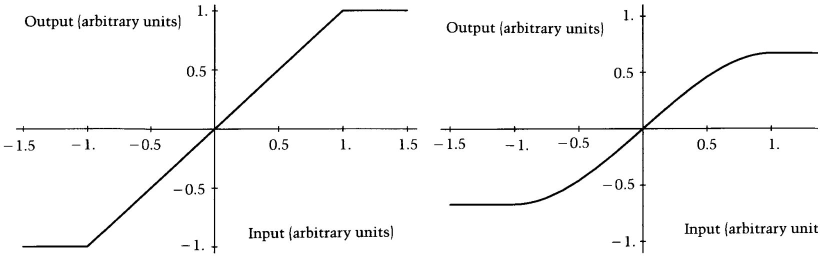
- To eliminate DC, add high-pass filter:
  - $y_n = a_0x_n + a_1x_{n-1} + b_1y_{n-1}$
- Need to provide continuous tuning:
  - Simple linear interpolation $y_n = c_0x_n + c_1x_{n-1}$
  - But this also produces attenuation (low-pass filter)
    - So adjust loop filter (FIR) to provide only the additional attenuation required
      - Might require compensating *boost* at higher frequencies
        - Don’t boost, sometimes higher frequencies will suffer

Tuning and Glissandi

- Use interpolation to control sub-sample length
- To glissando, slowly change c_0, c_1
- When one reaches 1, you can change the delay length by 1, flip c_0, c_1, and no glitch
- Need to change loop FIR filter when c_0, c_1 change
  - Change every sample? – Expensive
  - Change at control rate, e.g. 1000Hz? – creates artifact
  - Solution: change once per period so artifacts generate harmonics that are masked by string harmonics

Distortion

- Single note distortion just adds harmonics
- But: distortion of a sum of notes is not the sum of distorted notes



Soft Clipping Function

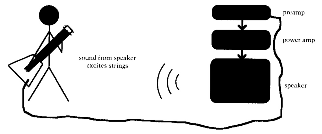
- F(x) =

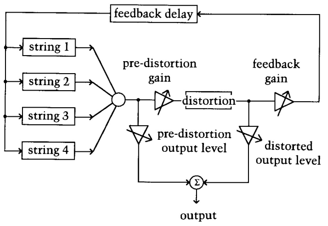
$2/3 \quad x \geq 1$

$x - x^3/3 \qquad -1 < x < 1$

$-2/3 \quad x \leq -1$

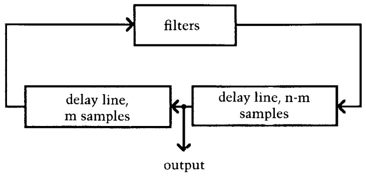
Feedback



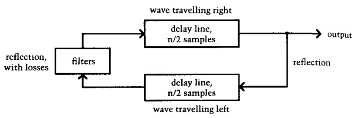


- Output can be pre- or post- distortion
- Will favor pitches and harmonics whose period matches feedback delay
- Possible to control exact onset and frequency of feedback

Pickup Position



Deriving output from a different point in the delay has little effect on the output.



Similar system, viewed as right-going and left-going waves on a string.

Pickup Position

Fig. 12

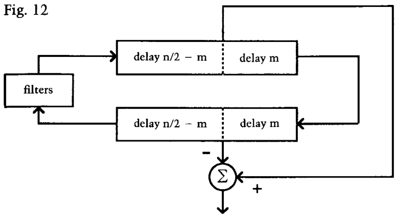
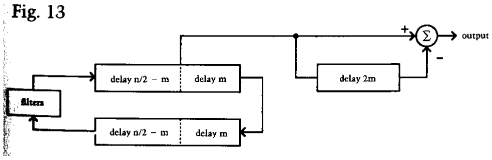
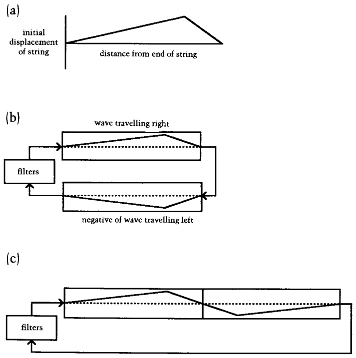


Fig. 13



Initializing the String



Additional Features

- Guitar body resonances
- Coloration and distortion of guitar amps
- Effects processors:
  - Distortion
  - Wah-wah pedals
  - Chorus…
- Reference: Charles R. Sullivan, "Extending the Karplus-Strong Algorithm to Synthesize Electric Guitar Timbres with Distortion and Feedback." Computer Music Journal, Vol. 14, No. 3, Fall 1990.

Analysis Example

- Estimation of loop filter based on decay of harmonics
- Exponential decay → straight lines on dB scale
- Slope relates to filter response
- Filter is fitted to measured data

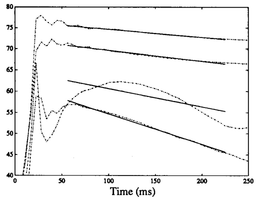


Fig. 7 *Temporal envelopes of the four lowest harmonics of a guitar tone and straight lines fits. The amplitude scale is in dB.*

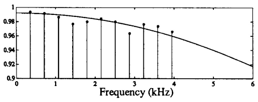


Fig. 8 *Estimated magnitude spectrum (circles) and magnitude response of a 1st-order IIR filter.*

Driving force

- In this model, after fitting filter to string recording,
- Inverse filter to obtain residual;
- Use residual to drive the string model to get realistic sound.
- Source: Karjalainen, Valimaki, and Janosy. “Towards High-Quality Sound Synthesis of the Guitar and String Instruments” in Proc. ICMC 1993.

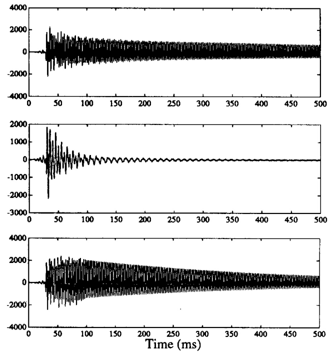


Fig. 9 *a) Original guitar tone, b) the inverse filtered signal, and c) the resynthesized signal.*

2-D Digital Waveguide Mesh

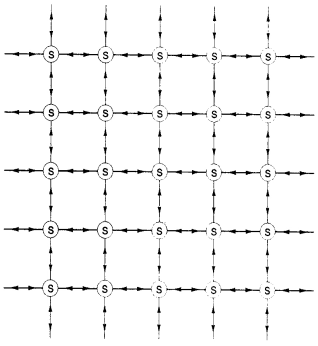
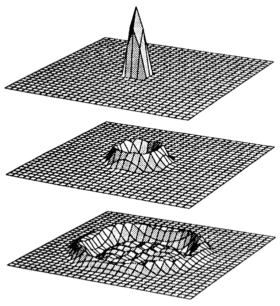


Figure 3. The 2-D Digital Waveguide Mesh



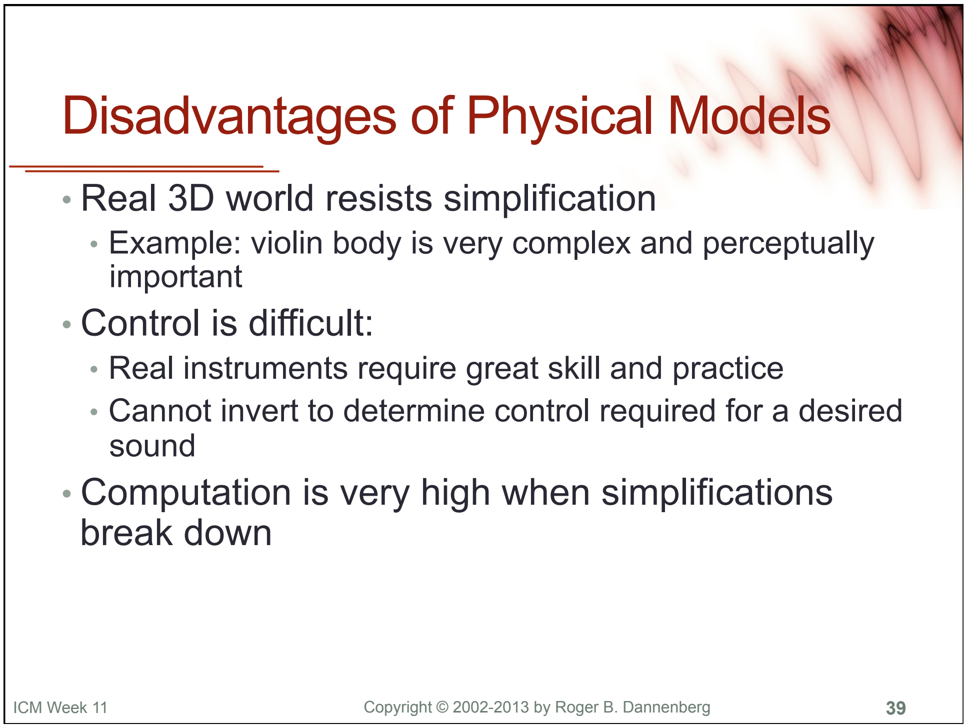
From: Van Duyne and Smith, “Physical Modeling with the 2-D Digital Waveguide Mesh,” in Proc. ICMC 1993.

Summary

- Bore or String modeled using delay
- Losses are "lumped" into a filter that closes the loop
- Non-linear element models driving force and generates oscillation
- Digital Waveguide offers efficient implementation – separates left- and right-going waves into 2 delays.

Advantages of Physical Modeling

- Non-linear and chaotic elements of instrument tend to arise naturally from models
- Models have relatively small set of controls
- Controls tend to be meaningful, intuitive
- Models tend to be modular, e.g. easy to add coupling between strings, refined loop filter, etc. to get better quality



Disadvantages of Physical Models

- Real 3D world resists simplification
  - Example: violin body is very complex and perceptually important
- Control is difficult:
  - Real instruments require great skill and practice
  - Cannot invert to determine control required for a desired sound
- Computation is very high when simplifications break down