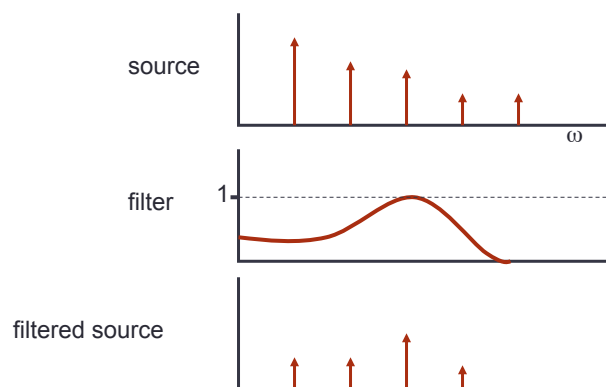


# FILTERS

Boosting and attenuating certain frequencies

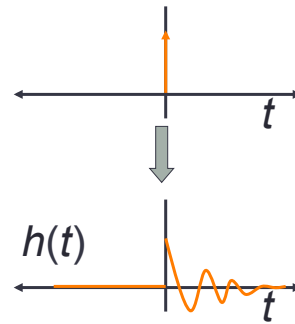
## Filters

- What is filtering?



## Some Intuition

- Suppose we know what a filter does to an impulse:
- This is called the *impulse response*  $h(t)$  of the filter



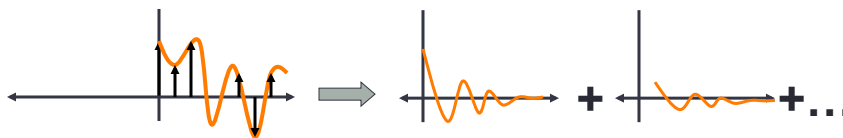
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3

## Superposition Principle

- Most filters are *linear time invariant* which implies that the superposition principle holds:  
 $\text{Response}(A+B) = \text{Response}(A) + \text{Response}(B)$
- Using superposition principle, we can sum up impulse responses to get response to signal

See: <http://www.jhu.edu/signals/convolve/>And <http://lpsa.swarthmore.edu/Convolution/Convolution3.html>

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4

## Convolution

$$y(t) = \int x(\tau)h(t - \tau)d\tau$$

- This expresses the infinite superposition of  $h$  scaled by the input signal  $x$ .
- This is the *convolution* of  $x$  and  $h$ :  $y = x * h$
- Convolution in the time domain is multiplication in the frequency domain:  $Y = X \times H$

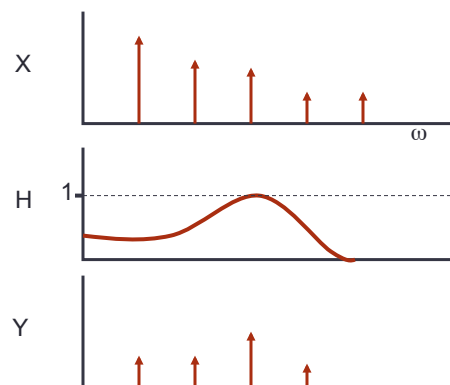
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5

## Back to where we started...

Multiplication is easier to think about, so that's why we like to describe filters as operating in the frequency domain



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## What about phase?

- Yes, filters can change phase
- In fact an all-pass filter has an amplitude response of 1 (unity gain at all frequencies)
- For the complete story, represent  $H$  as complex...

## Complex Frequency Response

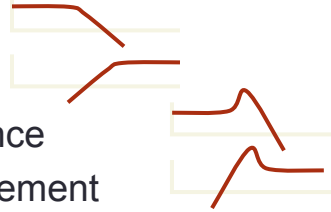
$$H(\omega) = U(\omega) + iV(\omega)$$

$$|H(\omega)| = \sqrt{U^2(\omega) + V^2(\omega)}$$

$$\angle H(\omega) = \arctan(V(\omega)/U(\omega))$$

## Filters in Nyquist

- LP – low-pass
- HP – high-pass
- RESON – resonance
- ARESON – complement



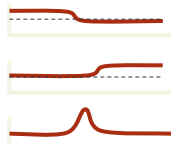
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9

## More Filters

- LOWPASS2, HIGHPASS2, BANDPASS2, NOTCH2, ALLPASS2– biquad filter variants
- EQ-LOWSHELF
- EQ-HIGHSHELF
- EQ-BAND
- LOWPASS[4,6,8], HIGHPASS[4,6,8]



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10

## Time-Varying Filters

- Time-varying filters are based on time invariant filters; simply change parameters
- Filter equation parameters are often expensive to compute, so change parameters at control rate
- Parameter changes can insert energy into filter system, and interpolated parameters can create instabilities – numerically difficult

## Examples

```
play reson(buzz(20, c3, const(0)),
           pwlv(100, 0.5, 3000, 1, 100),
           1000, 1) ~ 5
```



```
play reson(buzz(20, c3, const(0)),
           pwlv(100, 0.5, 3000, 1, 100),
           200, 1) ~ 5
```



```
play lp(buzz(20, c3, const(0)),
        pwev(20, 0.5, 3000, 1, 100)) ~ 5
```

